Homework 4

1. The probability that a student owns a microwave oven is 0.75, and that a student owns TV is 0.25. Probability that a student owns both a microwave and a TV is 0.16. Find the probability that a student owns either a microwave or a TV, but not both.

**Solution:** P(M) = 0.75, P(T) = 0.25, P (M intersection T) = 0.16

P(MUT) = P(M) + P(T) - P (M intersection T)

= 0.75 + 0.25 – 0.16

= 1.0 – 0.16

= 0.84

2. Five cards are drawn from a standard deck of cards without replacement. Find the probability of getting  
a. All red cards  
b. All diamonds  
c. All aces

**Solution:**  All red cards

(26/52) (25/51) (24/50) (23/49) (22/28) = 7893600/31187500200= 0.0253

All diamonds

(13/52) (12/51) (11/50) (10/49) (9/48) = 154440/ 31187500200 = 0.0049

All Aces

(4C4\*48C1)/52C5 = 0.000018

3. Suppose a person is randomly selected from a population of 1000 people with the distribution given below in the table.

Disease Status  
Age None Mild Moderate Severe Totals  
18 – 40 213 51 33 23 320  
Over 40 430 121 98 31 680  
Totals 643 172 131 54 1,000  
Find the probabilities of the following events that the person is  
a. P (Over 40)  
b. P (Mild and Over 40)  
c. P (Mild or Over 40)  
d. P (not Mild)  
e. P (Mild | Over 40)

**Solution:**

P (Over 40) = 680/1000 = 0.68

P (Mild and Over 40) = 121/1000 = 0.121

P (Mild or Over 40) = P(Mild) + P (Over 40) - P (Mild and Over 40)

= 0.172 + 0.68 – 0.121 = 0.731

P (not Mild) = 1 – P(Mild) = 1 – 0.172 = 0.828

P (Mild | Over 40) = P (Mild | Over 40 already happened)

= P (Mild ∩ Over40) / P(Over40)

= 0.121/0.68

= 0.1779

**Outcomes**  
3.1 You roll a four-sided die. What is the space of outcomes?

**Solution**: Space is either 1,2,3,4. Total 4 outcomes.

3.2 King Lear decides to allocate three provinces (1, 2, and 3) to his daughters (Goneril, Regan and Cordelia—read the  
book) at random. Each gets one province. What is the space of outcomes?

**Solution:** 3 daughters = G, R, C

3 provinces = 1,2,3

Sample Space = [G1R2C3,G1R3C2,G2C3R2,G2C2R3, G3R1C2,G3R2C1] = 6 possible ways for G1, similarly for R1 combination = 6 and C1 combination = 6 , total = 18

3.3 You randomly wave a flyswatter at a fly. What is the space of outcomes?  
Solution:

3.4 You read the book, so you know that King Lear had family problems. As a result, he decides to allocate two provinces  
to one daughter, one province to another daughter, and no provinces to the third. Because he’s a bad problem solver, he does  
so at random. What is the space of outcomes?

**Solution:** Sample Space = [(G12,R3) (R12,C3) ,(G13,R2),(R13,C2),(G23,R1),(R23,C1)

(G12,C3),(C12,R3),(G13,C2),(C13,R2),(G23,C1),(C23,R1)

(R12,G3),(C12,G3),(R13,G2),(C13,G2),(R23,G1),(C23,G1)

]

Where G12, R3 means G got 1,2 province and R got 3 and C didn’t get anything.

**The Probability of an Outcome**

3.5 You roll a fair four-sided die. What is the probability of getting a 3?

**Solution:** P (Getting 3) = ¼ = 0.25

3.6 You shuffle a standard deck of playing cards and draw a card. What is the probability that this is the king of hearts?

**Solution:** P (King of hearts) = 1/52

3.7 A roulette wheel has 36 slots numbered 1–36. Of these slots, the odd numbers are red and the even numbers are black. There are two slots numbered zero, which are green. The croupier spins the wheel, and throws a ball onto the surface; the ball bounces around and ends up in a slot (which is chosen fairly and at random). What is the probability the ball ends up in slot 2?

**Solution:** Sample Space = 36+ 2 = 38

P (Getting 2) = 1/38

**Events**  
3.8 At a particular University, 1/2 of the students drink alcohol and 1/3 of the students’ smoke cigarettes.  
(a) What is the largest possible fraction of students who do neither?

**Solution:** P(A) = ½ P(S) = 1/3

P(AUS) = P(A)+P(S) - P (A intersection S)

P(AUS)’ = 1 -[P(A) + P(S) - P (A intersection S)]

= 1 - [1/2+1/3 - P (A intersection S)]

= 1 - 5/6 + P (A intersection S)]

Hence P (who do neither) >=1/6

(b) It turns out that, in fact, 1/3 of the students do neither. What fraction of the students does both?

**Solution:** If P(AUS)’ = 1/3, then

1/3 = 1/6 + P (A intersection S)

P (A intersection S) = 1/3-1/6= 1/6

**Computing Probabilities by Counting Outcomes**

3.9 Assume each outcome in omh has the same probability. In this case, show P(E)=Number of outcomes in E/Total number of outcomes omh

**Solution:** Sample space = outcome1+outcome2……+outcomeN

Let e be the event which occurs for every outcome1,outcome2 …..outcomeN.

P(e1) = e1/outcome1, P(e2) = e2/outcome2  …. P(eN) = eN/outcomeN

P (E ) = number of outcomes in E/total outcomes in sample space

P(E) = count(P(e1), P(e2) …P(eN)/sample space

Hence P(E) = (Number of outcomes in E)/(Total number of outcomes)

3.10 You roll a fair four-sided die, and then a fair six-sided die. You add the numbers on the two dice. What is the probability the result is even?

**Solution:** Sample Space = 4\*6 = 24

P (Even on 4 sided) = 2/4= ½

P (Even on 6 sided) = 3/6 = ½

P(Even) = ½1/2 + ½1/2 = ½

3.11 You roll a fair 20-sided die. What is the probability of getting an even number?

**Solution:** Sample Space = 20

P(Even) = 10/20 = ½

3.12 You roll a fair five-sided die. What is the probability of getting an even number?

**Solution:** Sample Space = 5

P(Even) = 2/5

3.13 I am indebted to Amin Sadeghi for this exercise. You must sort four balls into two buckets. There are two white, one red and one green ball.

(a) For each ball, you choose a bucket independently and at random, with probability 1/2. Show that the probability each bucket has a colored ball in it is ½.

**Solution:** Sample Space 2W,1R,1G = 4 balls.

4Balls into 2 buckets = 2^4.

If we fix two colored balls in two different buckets then the total number of ways two white balls can be put in two buckets = 2^2 = 4

Therefore, the total number of ways the balls can be put in the buckets so that each bucket has colored ball = 4\*2 = 8

Hence the probability that each bucket has a colored ball = 8/16 = 1/2.

(b) You now choose to sort these balls in such a way that each bucket has two balls in it. You can do so by generating a permutation of the balls uniformly and at random, then placing the first two balls in the first bucket and the second two balls in the second bucket. Show that there are 16 permutations where there is one colored ball in each bucket.

**Solution:** Now we arrange the balls first then put the first two balls in the first bucket and the next two balls in the second bucket. The permutations are

RWGW, RWWG, WRWG, WRGW, GWRW, GWWR, WGWR, WGRW.

Total number of permutations such that each bucket has one colored ball = 8\*2 = 16

(c) Use the results of the previous step to show that, using the sorting procedure of that step, the probability of having a colored ball in each bucket is 2/3.

**Solution:** There 4 balls which can be ordered in 4! ways = 24

We can sort n balls with k bucket in K^N ways = 2^4 = 16

P (Colored balls in each bucket) = 16/24 = 2/3

(d) Why do the two sorting procedures give such different outcomes?

Solution: The answers for question B and C are different because we have considered the sorting of the balls in ordered mechanism with conditions applied for each B and C.

**The Probability of Events**  
3.14 You flip a fair coin three times. What is the probability of seeing HTH? (i.e., Heads, then Tails, then Heads)

**Solution:** Sample space = 2^3 = 8.

P(HTH) = 1/8 = 0.125 (same order)

P(HTH) = 3/8 = 0.375(without order)

3.15 You shuffle a standard deck of playing cards and draw a card.  
(a) What is the probability that this is a king?

**Solution:** P(King) = 4/52 = 2/26 = 1/13  
(b) What is the probability that this is a heart?

**Solution:** P(Heart) = 13/52 = ¼   
(c) What is the probability that this is a red card (i.e., a heart or a diamond)?

**Solution:** P (Heart Or Diamond) = 13+13C1/52C1 = 26/52 = ½

3.16 A roulette wheel has 36 slots numbered 1–36. Of these slots, the odd numbers are red and the even numbers are black.  
There are two slots numbered zero, which are green. The croupier spins the wheel, and throws a ball onto the surface; the  
ball bounces around and ends up in a slot (which is chosen fairly and at random).  
What is the probability the ball ends up in a green slot?

**Solution:** Total green slots = 2

Sample space = 36+2 = 38

P (Green slot) = 2/38 = 1/19

What is the probability the ball ends up in a red slot with an even number?

**Solution:** Red slot will never have even number hence its 0.

What is the probability the ball ends up in a red slot with a number divisible by 7?

**Solution:** Total red slots = 36+2=38

Number divisible by 7 {7,21,35} = 3

P (ball ends up in a red slot with a number divisible by 7) = 3/38

3.17 You flip a fair coin three times. What is the probability of seeing two heads and one tail?

**Solution:** Sample space = 2^3 = 8

P (2head and 1 tail) = 3C2 /8 = 3/8  
3.18 You remove the king of hearts from a standard deck of cards, then shuffle it and draw a card.  
(a) What is the probability this card is a king?

**Solution:** Sample space = 51

P(King) = 3/51= 1/17  
(b) What is the probability this card is a heart?

**Solution:** Sample space = 51

P(Hearts) = 12/51

3.19 You shuffle a standard deck of cards, then draw four cards.  
(a) What is the probability all four are the same suit?

**Solution:** Total cards = 52, each suit card = 13

Total cases = 52C4 = 270725

4 cards can be drawn from each suit = 4 \* 13C4 = 2860

P (All 4 same suit) = 2860/270725 = 2707252860​=0.0106

(b) What is the probability all four are red?

**Solution:** Sample space = 52C4

Red cards can be drawn as 26C4

P (All red) 26C4/52C4 = 0.0552

(c) What is the probability each has a different suit?

**Solution:** Sample space = 52C4.

All 4 cards are from different suits = 4\*13C1

P(Different Suit) = 4\*13C1/52C4 = 0.1055

3.20 You roll three fair six-sided dice and add the numbers. What is the probability the result is even?

**Solution:** Sample space = 6 ^3 = 216.

Case1: All values are even:

3∗3∗3=27

Case2: Two dices are odd and one of them is even.

3C2∗3∗3∗3=81

P(Even)= (81+27)/216=0.5

3.21 You roll three fair six-sided dice and add the numbers. What is the probability the result is even and not divisible by 20?

**Solution:** P(Even)\*1-P(Sum Divisible by 20)

The exhaustive number of cases = {1,2,3,4,5,6} = 6^3 = 216

Case1: All values are even:

3∗3∗3=27

Case2: Two dices are odd and one of them is even.

3C2∗3∗3∗3=81

P(Even) = 81+27/216 = 0.5  
Number divisible by 20 are 00, 20, 40, 60, 80…

P (Sum divisible by 20) = 0 since max sum we can get is 6+6+6= 18. And min sum = 1+1+1 = 3

P (Result is even and not divisible by 20) = P(Even) = 0.5

3.22 You shuffle a standard deck of cards, then draw seven cards. What is the probability that you see no aces?

**Solution:** Sample space = 52C7 = 133784560

P(No Aces) = 1 - P(At least 1 Ace)

Drawing one ace and six non-aces: Number of ways = 4C1\* 48C6 = 4 \* 12271512 = 49086048 ways.

Drawing two aces and five non-aces: Number of ways 4C2\* 48C5 = 6 \* 1712304 = 10273824 ways.

Drawing three aces and four non-aces: Number of ways =4C3 \* 48C4 = 4 \* 194580 = 778320 ways.

Drawing four aces and three non-aces: Number of ways = 4C4\* 48C3 = 1 \* 17296 = 17296 ways.

Total ways to draw at least one ace = 49086048 + 10273824 + 778320 + 17296 = 60046988 ways.

P (No Aces) = 1-(60046988 / 133784560) = 0.5507

3.23 Show that P.A .B [ C// D P.A/ P.A \ B/ P.A \ C/ C P.A \ B \ C/.

**Solution:**

Let’s use A−(B∪C) = A−(B∪C) =A∩(B∪C)’ = A∩(B∪C)’=A∩(B’∩C’) = A∩(B’∩C’) =(A∩B’) ∩(A∩C’)

P(A∩(B’∩C’)) = P((A∩B’) ∩(A∩C’))= P(A)−P(A∩B)−P(A∩C)+P(A∩B∩C)=P(A−(B∪C))P(A)−P(A∩B)−P(A∩C)+P(A∩B∩C).

3.24 You draw a single card from a standard 52 card deck. What is the probability that it is red?

**Solution:** Sample Space = 52

Red cards = 26

P (Red Cards) = 26/52 = 1/2  
3.25 You remove all heart cards from a standard 52 card deck, then draw a single card from the result.  
(a) What is the probability that the card you draw is a red king?

**Solution:** Sample Space = 39

P(Red King) =(P Diamond King) = 1/39  
(b) What is the probability that the card you draw is a spade

**Solution:** Sample Space = 39

Number of Spades = 13

P(Spade) = 13/39 = 1/3

**Permutations and Combinations**  
3.26 You shuffle a standard deck of playing cards, and deal a hand of 10 cards. With what probability does this hand have five red cards?

**Solution:**  Sample space = 52C10 = 15820024220.

5 red cards can be selected as 13+13C5 = 26C5 = 65780.

P(5 red cards) = 65780/15820024220 = 0.000004158.

3.27 Magic the Gathering is a popular card game. Cards can be land cards, or other cards. We consider a game with two players. Each player has a deck of 40 cards. Each player shuffles their deck, then deals seven cards, called their hand.  
(a) Assume that player one has 10 land cards in their deck and player two has 20. With what probability will each player have four lands in their hand?

Solution: Total cards = 40. Sample space = 40C7 = 18643560.

P1 land cards = 10, P2 land cards = 20.

P (p1 have 4 lands) = 10C4\*30C3/18643560 = (210 \* 4060)/18643560 = 0.457

P (p2 have 4 lands) = 20C4 \* 20C3/18643560 = 4845\* 1140/18643560 = 0.296

(b) Assume that player one has 10 land cards in their deck and player two has 20. With what probability will player one has  
two lands and player two have three lands in hand?

**Solution:** P (p1 have 2 lands) = 10C2\*30C5/40C7 = 0.343

P (p2 have 3 lands) = 20C3\*20C4/40C7 = 0.296  
(c) Assume that player one has 10 land cards in their deck and player two has 20. With what probability will player two have more lands in hand than player one?

**Solution:**  Sample space = 40C7

Let P1 draws all the lands in hands = 10C7

Let P2 draws all the lands in hand = 20C7

P (p1 have all 7 lands) = 10C4/40C7= 0.0000064356

P (p2 have all 7 lands) = 20C4/40C7 = 0.004158004

P (p2 have more lands) = 0.004158004 - 0.0000064356 = 0.0041515675

3.28 The previous exercise divided Magic the Gathering cards into lands vs. other. We now recognize four kinds of cards: land, spell, creature and artifact. We consider a game with two players. Each player has a deck of 40 cards. Each player  
shuffles their deck, then deals seven cards, called their hand.  
(a) Assume that player one has 10 land cards, 10 spell cards, 10 creature cards and 10 artifact cards in their deck. With what probability will player one has at least one of each kind of card in hand?

**Solution:** P (At least 1 card of each type) = 1 - P (not having at least one of each kind)

P (No land card) = 30C7/40C7 = 2035800/ 18643560 = 0.1091 = P(A)

P (No spell card) = 30C7/40C7 = 2035800/ 18643560 = 0.1091 = P(B)

P (No creature card) = 30C7/40C7 = 2035800/ 18643560 = 0.1091 = P(C)

P (No artifact card) = 30C7/40C7 = 2035800/ 18643560 = 0.1091 = P(D)

P(A ∪ B ∪ C ∪ D) = P(A) + P(B) + P(C) + P(D) - P(A ∩ B) - P(A ∩ C) - P(A ∩ D) - P(B ∩ C) - P(B ∩ D) - P(C ∩ D) + P(A ∩ B ∩ C) + P(A ∩ B ∩ D) + P(A ∩ C ∩ D) + P(B ∩ C ∩ D) - P(A ∩ B ∩ C ∩ D)

P(A ∩ B) = (10C7\*10C7)/40C7 = 120\*120/18643560 = 0.0007723

P(A ∩ C) = (10C7\*10C7)/40C7 = 120\*120/18643560 = 0.0007723

P(A ∩ D) = (10C7\*10C7)/40C7 = 120\*120/18643560 = 0.0007723

P(B ∩ C) = (10C7\*10C7)/40C7 = 120\*120/18643560 = 0.0007723

P(B ∩ D) = (10C7\*10C7)/40C7 = 120\*120/18643560 = 0.0007723

P(C ∩ D) = (10C7\*10C7)/40C7 = 120\*120/18643560 = 0.0007723

P(A ∩ B ∩ C) =10C7\*10C7\*10C7\*10C7/40C7 = 0.00182257

P(A ∩ B ∩ D) =10C7\*10C7\*10C7/40C7 = 0.00182257

P(A ∩ C ∩ D) =10C7\*10C7\*10C7/40C7 = 0.00182257

P(B ∩ C ∩ D)= 10C7\*10C7\*10C7/40C7 = 0.00182257

P(A ∩ B ∩ C ∩ D)= 120^4\*4/40C7 = 0.2707

So 0.1091 \*4 - (0.0007723\*6) +(0.00182257\* 4) - 0.2707

= 0.4364 – 0.0046338 + 0.00729028 - 0.2707

= 0.44369028 - 0.2753338

= 0.16835648

P (Having 1 card of each land) = 1 - 0.16835648 = 0.83164352

(b) Assume that player two has 20 land cards, 5 spell cards, 7 creature cards and 8 artifact cards in their deck. With what probability will player two have at least one of each kind of card in hand?

**Solution:**

1. Probability of having at least one land, one spell, one creature, and one artifact card in hand:
   * Number of ways to choose at least 1 land card from Player two's 20 land cards = 1−C(20,0)C(20,7)1−C(20,7)C(20,0)​
   * Number of ways to choose at least 1 spell card from Player two's 5 spell cards = 1−C(5,0)C(5,7)1−C(5,7)C(5,0)​
   * Number of ways to choose at least 1 creature card from Player two's 7 creature cards = 1−C(7,0)C(7,7)1−C(7,7)C(7,0)​
   * Number of ways to choose at least 1 artifact card from Player two's 8 artifact cards = 1−C(8,0)C(8,7)1−C(8,7)C(8,0)​

Probability = (1−C(20,0)C(20,7))⋅(1−C(5,0)C(5,7))⋅(1−C(7,0)C(7,7))⋅(1−C(8,0)C(8,7))(1−C(20,7)C(20,0)​)⋅(1−C(5,7)C(5,0)​)⋅(1−C(7,7)C(7,0)​)⋅(1−C(8,7)C(8,0)​)

1. Probability of having at least one land, one spell, one creature, but no artifact card in hand:
   * Number of ways to choose at least 1 land card from Player two's 20 land cards = 1−C(20,0)C(20,7)1−C(20,7)C(20,0)​
   * Number of ways to choose at least 1 spell card from Player two's 5 spell cards = 1−C(5,0)C(5,7)1−C(5,7)C(5,0)​
   * Number of ways to choose at least 1 creature card from Player two's 7 creature cards = 1−C(7,0)C(7,7)1−C(7,7)C(7,0)​
   * Number of ways to choose 7 cards from Player two's 8 artifact cards = C(8,7)C(8,7)

Probability = (1−C(20,0)C(20,7))⋅(1−C(5,0)C(5,7))⋅(1−C(7,0)C(7,7))⋅C(8,7)C(8,7)(1−C(20,7)C(20,0)​)⋅(1−C(5,7)C(5,0)​)⋅(1−C(7,7)C(7,0)​)⋅C(8,7)C(8,7)​

1. Probability of having at least one land, one spell, one artifact, but no creature card in hand:
   * Number of ways to choose at least 1 land card from Player two's 20 land cards = 1−C(20,0)C(20,7)1−C(20,7)C(20,0)​
   * Number of ways to choose at least 1 spell card from Player two's 5 spell cards = 1−C(5,0)C(5,7)1−C(5,7)C(5,0)​
   * Number of ways to choose 7 cards from Player two's 7 creature cards = C(7,7)C(7,7)
   * Number of ways to choose at least 1 artifact card from Player two's 8 artifact cards = 1−C(8,0)C(8,7)1−C(8,7)C(8,0)​

Probability = (1−C(20,0)C(20,7))⋅(1−C(5,0)C(5,7))⋅C(7,7)C(7,7)⋅(1−C(8,0)C(8,7))(1−C(20,7)C(20,0)​)⋅(1−C(5,7)C(5,0)​)⋅C(7,7)C(7,7)​⋅(1−C(8,7)C(8,0)​)

1. Probability of having at least one land, one creature, one artifact, but no spell card in hand:
   * Number of ways to choose at least 1 land card from Player two's 20 land cards = 1−C(20,0)C(20,7)1−C(20,7)C(20,0)​
   * Number of ways to choose 7 cards from Player two's 5 spell cards = C(5,7)C(5,7)
   * Number of ways to choose at least 1 creature card from Player two's 7 creature cards = 1−C(7,0)C(7,7)1−C(7,7)C(7,0)​
   * Number of ways to choose at least 1 artifact card from Player two's 8 artifact cards = 1−C(8,0)C(8,7)1−C(8,7)C(8,0)​

Probability = (1−C(20,0)C(20,7))⋅C(5,7)C(5,7)⋅(1−C(7,0)C(7,7))⋅(1−C(8,0)C(8,7))(1−C(20,7)C(20,0)​)⋅C(5,7)C(5,7)​⋅(1−C(7,7)C(7,0)​)⋅(1−C(8,7)C(8,0)​)

(c) Assume that player one has 10 land cards, 10 spell cards, 10 creature cards and 10 artifact cards in their deck;. and player two has 20 land cards, 5 spell cards, 7 creature cards and 8 artifact cards in their deck. With what probability will at least  
one of the players have at least one of each kind card in hand?

**Solution:**

1. Calculate the probability that Player one does not have at least one of each kind of card in hand:

To calculate this, we'll find the probability that Player one's hand does not contain at least one land, one spell, one creature, and one artifact card.

* + Probability that Player one's hand doesn't have at least one land, one spell, one creature, and one artifact card:
    - Calculate the probability for each type of card not being in Player one's hand (complements):
      * Probability that Player one's hand doesn't have a land card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
      * Probability that Player one's hand doesn't have a spell card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
      * Probability that Player one's hand doesn't have a creature card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
      * Probability that Player one's hand doesn't have an artifact card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
  + Now, calculate the combined probability that Player one's hand doesn't have at least one of each kind:
    - Probability that Player one doesn't have at least one of each kind = Probability that Player one's hand doesn't have a land card \* Probability that Player one's hand doesn't have a spell card \* Probability that Player one's hand doesn't have a creature card \* Probability that Player one's hand doesn't have an artifact card

1. Calculate the probability that Player two does not have at least one of each kind of card in hand:

Similar to Player one, we'll find the probability that Player two's hand does not contain at least one land, one spell, one creature, and one artifact card.

1. Calculate the combined probability that neither player has at least one of each kind:
   * Probability that neither player has at least one of each kind = Probability that Player one doesn't have at least one of each kind \* Probability that Player two doesn't have at least one of each kind
2. Finally, calculate the probability that at least one player has at least one of each kind by subtracting the probability from step 3 from 1:
   * Probability that at least one player has at least one of each kind = 1 - Probability that neither player has at least one of each kind

Now, let's calculate these probabilities step by step:

1. Probability that Player one doesn't have at least one of each kind:
   * Probability that Player one doesn't have a land card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
   * Probability that Player one doesn't have a spell card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
   * Probability that Player one doesn't have a creature card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
   * Probability that Player one doesn't have an artifact card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​

Now, calculate the combined probability:

Probability that Player one doesn't have at least one of each kind = Probability that Player one doesn't have a land card \* Probability that Player one doesn't have a spell card \* Probability that Player one doesn't have a creature card \* Probability that Player one doesn't have an artifact card

1. Probability that Player two doesn't have at least one of each kind:
   * Probability that Player two doesn't have a land card: 1−C(30,7)C(40,7)1−C(40,7)C(30,7)​
   * Probability that Player two doesn't have a spell card: 1−C(25,7)C(40,7)1−C(40,7)C(25,7)​
   * Probability that Player two doesn't have a creature card: 1−C(33,7)C(40,7)1−C(40,7)C(33,7)​
   * Probability that Player two doesn't have an artifact card: 1−C(32,7)C(40,7)1−C(40,7)C(32,7)​

Now, calculate the combined probability:

Probability that Player two doesn't have at least one of each kind = Probability that Player two doesn't have a land

3.29 You take a standard deck of 52 playing cards and shuffle it. Compute the probability that, in the shuffled deck, there is at least one pair of cards following one another in increasing order (i.e., a 2 followed by a 3, or a 3 followed by a 4, etc.). This isn’t particularly easy, but the probability is higher than most people realize; you can surprise your friends and make money with this information

**Solution:**

For each suit, there are 12 possibilities for consecutive pairs (2-3, 3-4, ..., 10-Jack, Jack-Queen, Queen-King, King-Ace).

Since there are 4 suits, there are a total of 4 \* 12 = 48 ways to create consecutive pairs within a single suit.

The total number of ways to have pairs of consecutive cards in the deck is 48 (pairs within a single suit) \* 52 (positions for the first card) \* 51 (positions for the second card).

The total number of ways to arrange 52 cards in a deck is 52! (52 factorial), which is the total number of possible outcomes.

Now, we can calculate the probability that no consecutive pairs exist and then subtract it from 1 to get the probability that at least one pair exists:

Probability of no consecutive pairs = (52! - (48 \* 52 \* 51)) / 52!

Probability of at least one consecutive pair = 1 - Probability of no consecutive pairs=0.7551

P (Pair following other) = 1 - 0.7551 = 0.2449